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# REPLY TO THE COMMENTS BY YUAN WENXUE, ZHOU LANTING AND M. XIAOQING ON THE BALLISTIC PENETRATION THEORY OF AWERBUCH AND BODNER

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The author appreciates the attention that the authors of the preceding "Comments" have given to the paper by Awerbuch and himself on modeling the ballistic perforation process[1]. It seems that those authors have tended to examine[1] in terms of their own recently published model of the process[2], leading to the resulting criticism. In doing so, they appear to have misinterpreted the model presented in[1], and have not properly applied the principles of mechanics in relation to that model.

Firstly, it should be noted that the model described in [1] is a considerably simplified one-dimensional (1D) concept of the complex process of ballistic perforation. The actual inelastic process zone surrounding the projectile is essentially two dimensional (2D) in nature (for normal impact). It is therefore difficult to obtain a 1D idealization that incorporates most of the important features of the perforation process. The idealized model should, of course, be as physically consistent as possible and lead to a proper set of governing equations from the mechanics viewpoint.

The main assumption for the first two stages of the perforation process described in [1] is that an extent of target material forward of the projectile moves together with the projectile, and that the rate of increase of that zone is the projectile velocity. Material in the inelastic process zone is taken to be at the compressive strength (flow stress) and to have acquired kinetic energy corresponding to the projectile velocity. The criticism of the model is directed at consequences of this assumption, since it implies a "piling up" of material in front of the projectile, and gives rise to an additional inertial term in the equation of motion.

A more accurate physical picture of the mechanism is that the material in the inelastic process zone moves not only longitudinally but radially as well, and that the zone undergoes radial expansion. The added mass of target material in the simplified 1D model of [1], $\rho Ax$, can be viewed as an approximation to the target mass in a 2D formulation that is effectively moving with the projectile velocity. A 2D analysis, e.g. [3], considers both longitudinal and radial motions in an inelastic process zone wider than the projectile, and also treats rearward expulsion of the displaced target material. Numerical exercises based on the 2D model of [3] indicate that the longitudinal extent of the inelastic zone forward of the projectile becomes essentially constant after a very short transient period, so that the front of the zone moves at the projectile velocity. These results tend to support the main assumption of [1], which is difficult to graphically interpret within a 1D geometrical format.

The extra inertial term in the equation of motion, $\rho AV^2$, arises directly from the main assumption of the mechanism involving a variable added mass. A clear account of variable mass problems appears in [4] as well as other books in basic mechanics. Either the analysis given by [1] can be adopted, or eqn (5), p. 29, of [4] which states that if $m$ is the current mass of a body subjected to a force $F$, then

$$m\dot{\mathrm{V}} = F + \dot{m}\mathrm{V}_{\mathrm{rel}}, \quad (1)$$

where $\dot{m}$ is the convective mass rate and $\mathrm{V}_{\mathrm{rel}}$ is its velocity relative to the body under observation, measured positive in the same sense as V. In the case under consideration, $\dot{m} = \rho A V$, $\mathbf{V}_{rel} = -V$, and $m = m_0 + \rho A x$, where $x$ is the extent of moving target material. The equation for stage 1 of [1] is therefore

$$(m_0 + \rho A X) \frac{dV}{dt} + \rho A V^2 = F = -F_c - F_i, \tag{2}$$

where $F_c = \sigma_c A$ and $F_i = (\frac{1}{2}) K \rho A V^2$, while a shearing force $F_s$ also acts during stage 2 of [1].

An interpretation of the term $\rho A V^2$ in the equation of motion is that it is due to the convective transfer of material from the target to the combined target and added mass and therefore acts as an effective resisting force. This force is in addition to the usual inertial force $F_i$ which is required to bring the velocity of the target mass from rest to the velocity $V$. It is noted that an alternative 1D model, with a constant length process zone and radial ejection (relative to the projectile) of the ingested target material, would also lead to the extra inertial term by control volume arguments. If the extra inertial term were not included in the equation of motion, then a radial restraint effect must be hypothesized, and the usual procedure is to increase the material strength by an arbitrary factor to account for it. In eqn (2), the actual material strength is used in $F_c$.

The present writer therefore does not agree with the criticism presented in the preceding note. In particular, the analogy made between the mechanism described in[1] and 1D plastic wave theory is inappropriate, since that theory is not relevant to the suggested model. The writers of the "Comments" also seem to have misinterpreted the model in applying to it the principles of basic mechanics. In summary, the equations of [1] are held to be correct and appropriate for the 1D model described in the paper. A recent paper[5] indicates that those equations could be integrated to give an energy expression similar in form to that developed by Recht and Ipson[6]. Practical limitations of the 1D model of [1] and the need for some empirical information are obvious. Nevertheless, they can serve as a simple method for obtaining approximate results for perforation of moderately thick plates [$h \sim 0.5$ to $4R$ (projectile radius)].

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